



## About distribution of own waves in the dissipative layered cylindrical bodies interacting with Wednesday

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### General Note

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### ABSTRACT

The paper deals with the propagation of its own wave dissipative layered cylindrical bodies interacting with deformable (viscoelastic) media. The dynamic behavior of cylindrical bodies is described by equations of continuum mechanics. The spectral problem is reduced to solving a system of ordinary differential equations

of the first order with variable complex coefficients. Solutions of differential equations are expressed by a special cylindrical Bessel functions and Hankel. The frequency equation is solved numerically by Mueller and Gauss. The change of own frequency and phase velocity as a function of the wave number. For inhomogeneous dissipative mechanical systems found non-monotonic dependence of the imaginary part of the phase velocity of the wave numbers.

**Keywords:** own waves, dissipative body, Wednesday, spectral problem, the natural frequency and phase velocity.

## 1. INTRODUCTION

Regularities of distribution of indignations in continuous environments are of considerable interest to many fields of science and technology [1,2]. In this work, unlike known, distribution of own waves in layered dissipatively heterogeneous (elements of mechanical system have different rheological properties) cylindrical bodies is considered [3,4,5].

The analysis of distribution of waves in elastic medium is based on an assumption about justice of Hooke's law according to which tension in this point of the environment in timepoints is pro rata to deformations in the same timepoints [6,7]. The fact that energy of a series of waves or an impulse remains to a constant is a consequence of this assumption. As a rule, influence of these "no - gukian" of tension for metals and metal alloys isn't enough, however they are of great importance for rubber of similar and construction materials which find essential temporary effects under loading [8]. Also different viscoelastic (rheological) properties of dissipatively heterogeneous bodies distribution and attenuations of waves significantly influences the law [9,10,11].

## 2. STATEMENT OF THE PROBLEM AND METHODS OF SOLUTION

In cylindrical system of coordinates  $\{ r, \theta, z \}$  the distribution of own waves in an isotropic viscoelastic body (V) consisting of piecewise and uniform cylindrical bodies of  $V_k$  is considered ( $k=1, N$ ), shipped on homogeneous viscoelastic environment (fig. 1).

Linear the equation of the movtion of mechanical system in a vector form in the absence of volume forces takes a form:

$$\tilde{\mu}_k \nabla^2 \vec{u} + (\tilde{\lambda}_k + \tilde{\mu}_k) \text{graddiv} \vec{u} = \rho_k \frac{\partial^2 \vec{u}}{\partial t^2}, \quad (k=1, 2, 3, \dots, N.) \quad (1)$$

where

$$\begin{aligned}\tilde{\lambda}_\kappa f(t) &= \lambda_{\kappa 0} \left[ f(t) - \int_{-\infty}^t R_{\lambda\kappa}(t-\tau) f(t) d\tau \right], \\ \tilde{\mu}_\kappa f(t) &= \mu_{\kappa 0} \left[ f(t) - \int_{-\infty}^t R_{\mu\kappa}(t-\tau) f(t) d\tau \right],\end{aligned}\quad (2)$$

$f(t)$ —arbitrary function of time;  $R_{\lambda\kappa}(t-\tau)$  and  $R_{\mu\kappa}(t-\tau)$ —relaxation kernel;  $\lambda_{\kappa 0}, \mu_{\kappa 0}$ —instant elastic moduli ( $\kappa=1, \dots, N$ );  $\vec{u}$ —displacement vector;  $\rho_\kappa$ —density of the medium,  $\kappa$ —ordinal number of layers. Between the layers is put hard conditions or sliding contact [12]

$$\begin{aligned}r = a_\kappa : \quad \sigma_{rr\kappa} &= \sigma_{rr(\kappa+1)}; \quad \sigma_{r\theta\kappa} = \sigma_{r\theta(\kappa+1)}; \quad \sigma_{rz\kappa} = \sigma_{rz(\kappa+1)}; \\ u_\kappa &= u_{\kappa+1}; \quad \mathcal{G}_\kappa = \mathcal{G}_{\kappa+1}; \quad w_\kappa = w_{\kappa+1}.\end{aligned}\quad (3, a)$$

If the outer surface is free from multilayer cylinder stresses, whereas

$$r = a_{\kappa+1} : \quad \sigma_{rr(\kappa+1)} = 0; \quad \sigma_{r\theta(\kappa+1)} = 0; \quad \sigma_{rz(\kappa+1)} = 0. \quad (3, b)$$

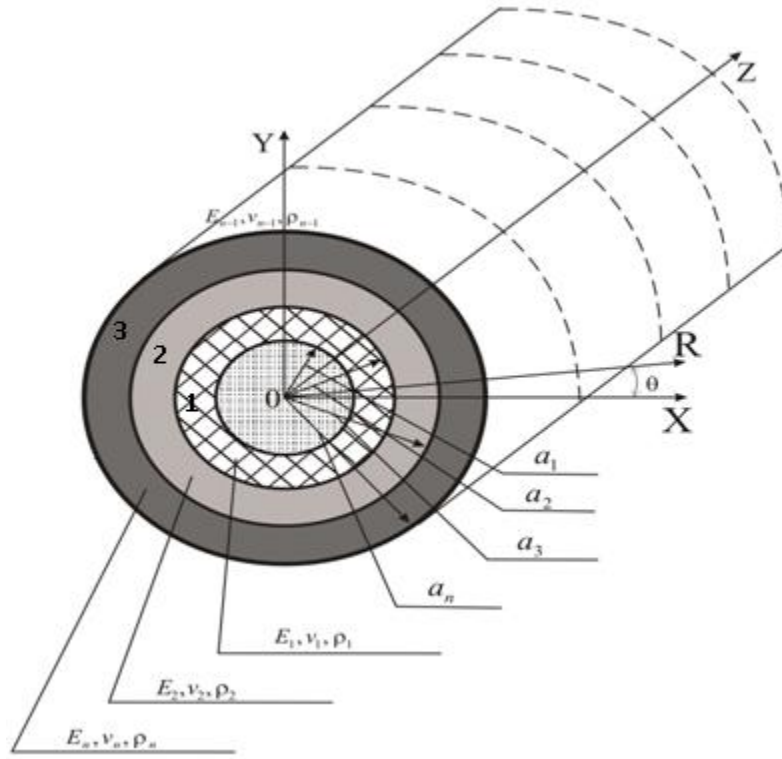
If the displacement vector presented in the form of potential and solenoidal parts

$$\vec{u} = \text{grad } \varphi + \text{rot } \vec{\psi},$$

the wave equation have the form

$$\begin{aligned}\Delta \phi_k - \frac{1}{c_{pj}^2} \frac{\partial^2 \phi_k}{\partial t^2} &= 0; \\ \Delta \psi_{zk} - \frac{1}{c_{sj}^2} \frac{\partial^2 \psi_{zk}}{\partial t^2} &= 0; \\ \Delta \psi_{rk} - \frac{\psi_{rk}}{r^2} + \frac{2}{r^2} \frac{\partial \psi_{\theta k}}{\partial \theta} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{rk}}{\partial t^2} &= 0; \\ \Delta \psi_{\theta k} - \frac{\psi_{\theta k}}{r^2} + \frac{2}{r^2} \frac{\partial \psi_{rk}}{\partial \theta} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{\theta k}}{\partial t^2} &= 0; \\ \Delta &= \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}; \\ u_{rk} &= \frac{\partial \phi_k}{\partial r} + \frac{1}{r} \frac{\partial \psi_{zk}}{\partial \theta} - \frac{\partial \psi_{\theta k}}{\partial z}; \\ u_{\theta k} &= \frac{1}{r} \frac{\partial \phi_k}{\partial \theta} + \frac{\partial \psi_{zk}}{\partial z} - \frac{\partial \psi_{\theta k}}{\partial r}; \\ u_{zk} &= \frac{\partial \phi_k}{\partial r} + \frac{\partial \psi_{\theta k}}{\partial r} + \frac{\partial \psi_{zk}}{\partial r} - \frac{1}{r} \frac{\partial \psi_{rk}}{\partial \theta}, \quad k = 1, 2, \dots, N.\end{aligned}\quad (4)$$

where  $\varphi$  – potential of longitudinal waves;  $\vec{\psi} (\psi_r, \psi_\theta, \psi_z)$  – potential transverse waves.



**Figure 1** Design scheme of piecewise-homogeneous cylindrical body

In the construction of representations for motion vector components  
a multilayer cylinder

$$0 < r_1 \leq a_1; \quad a_1 \leq r_2 \leq a_2; \dots a_{k-1} \leq r_k \leq a_k; \quad a_{n-1} \leq r_n \leq \infty \quad (k = 1, 2, \dots, N) \quad |z_1| < \infty$$

we proceed from the equation (4) for the scalar  $\phi(r, \theta, z, t)$  and the vector  $\vec{\psi}(r, \theta, z, t)$  potentials. The geometry of the object and the natural assumption about the nature of wave motion along the axis OZ allow largely to predict the shape of the desired scalar and vector functions. They should represent waves traveling along the OZ axis. The solutions of (4) in the form:

$$\left. \begin{aligned} \phi_k(r, \theta, z, t) &= \sum_{n=0}^{\infty} \phi_n(\alpha_k r) \begin{Bmatrix} \cos n\theta \\ -\sin n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\omega t}; \\ \psi_{rk}(r, \theta, z, t) &= \sum_{n=0}^{\infty} \psi_{nr}(\beta_k r) \begin{Bmatrix} \sin n\theta \\ -\cos n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\omega t}; \\ \psi_{\theta k}(r, \theta, z, t) &= \sum_{n=0}^{\infty} \psi_{n\theta}(\beta_k r) \begin{Bmatrix} \cos n\theta \\ -\sin n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\omega t}; \\ \psi_{zk}(r, \theta, z, t) &= \sum_{n=0}^{\infty} \psi_{nz}(\beta_k r) \begin{Bmatrix} \sin n\theta \\ \cos n\theta \end{Bmatrix} e^{\pm i\gamma_p z} e^{-i\omega t}; \end{aligned} \right\} \quad (5)$$

where  $n$  - integer;  $\gamma_{pk}$  - constant wave propagation;  $\omega$  - natural frequency;  $r = \frac{r_1}{a_0}$ ,  $z = \frac{z_1}{a_0}$ .

At infinity ( $r \rightarrow \infty$ ) put Sommerfeld conditions for each component. Substituting (5) into (4) we obtain the following ordinary differential equation:

$$\begin{aligned} \frac{d^2 \phi_k}{dr^2} + \frac{1}{r} \frac{d\phi_k}{dr} + \left( \alpha_k^2 - \frac{n^2}{r^2} \right) \phi_k &= 0; \\ \frac{d^2 \psi_{zk}}{dr^2} + \frac{1}{r} \frac{d\psi_{zk}}{dr} + \left( \beta_k^2 - \frac{n^2}{r^2} \right) \psi_{zk} &= 0; \end{aligned} \quad (6)$$

$$\begin{aligned} \frac{d^2 \psi_{\theta k}}{dr^2} + \frac{1}{r} \frac{d\psi_{\theta k}}{dr} + \frac{1}{r^2} (-n^2 \psi_{\theta k} + 2n \psi_{\theta k} - \psi_{\theta k}) \beta^2 \psi_{\theta k} &= 0; \\ \frac{d^2 \psi_{rk}}{dr^2} + \frac{1}{r} \frac{d\psi_{rk}}{dr} + \frac{1}{r^2} (-n^2 \psi_{rk} + 2n \psi_{\theta k} - \psi_{rk}) \beta^2 \psi_{rk} &= 0; \end{aligned}$$

$$\text{where } \alpha_k^2 = \frac{\Omega_k^2}{\gamma_k^2} - \gamma_p^2; \quad \beta_k^2 = \Omega_k^2 - \gamma_p^2; \quad \Omega_k = \frac{\omega \alpha_k}{c_{sk}}; \quad \gamma_k^2 = \frac{2(1 - \nu_k)}{1 - 2\nu_k}.$$

The first two equations in (6) the following solutions for  $k = 1$  of the first (solid) and the outer cylinder ( $k=N$ ):

$$\phi_k(r) = \begin{cases} F_{nk} J_n(\alpha_k r), & k = 1; \\ F'_{nk} H_n^{(1)}(\alpha_k r), & k = N; \end{cases} \quad (7,a)$$

$$\psi_z(r) = \begin{cases} M_{1k} J_n(\beta_k r), & k = 1; \\ M'_{1k} H_n^{(1)}(\beta_k r), & k = N; \end{cases} \quad (7,b)$$

where  $J_n(\alpha_k r)$  - Bessel function  $n$  - th order.  $H_n^{(1)}(\beta_k r)$  - Hankel function of the first kind  $n$  - th order.

The decision of the other two equations in (6) is also expressed in terms of Bessel functions and Hankel:

$$\psi_{rk}(r) = \begin{cases} L_{1nk} J_{n-1}(\beta_k r) + L_{2nk} J_{n+1}(\beta_k r), & k = 1; \\ L'_{1nk} H_{n-1}^{(1)}(\beta_k r) + L'_{2nk} H_{n+1}^{(2)}(\beta_k r), & k = N; \end{cases} \quad (8)$$

Solutions (7a), (7b) and (8) of the system of differential equations (6) will contain a  $6k-2$  arbitrary constants. I take this opportunity to go to a large extent arbitrary values when choosing a permanent believe further  $L_{1k} = L'_{1k} = 0$ , i.e.  $\psi_{rk} = \psi_{\theta k}$ .

Moving  $k$  cylinder is expressed in terms of Bessel and Neumann functions of  $n$ -th order complex argument

$$\begin{aligned}
u_{r\kappa} &= \sum_{n=0}^{\infty} \left\{ \gamma_{\kappa} [A_{1\kappa n} J'_n(\gamma_{1\kappa} r) + A_{2\kappa n} Y'_n(\gamma_{1\kappa} r)] + \frac{n}{r} [A_{3\kappa n} J_n(\gamma_{2\kappa} r) + A_{4\kappa n} Y_n(\gamma_{2\kappa} r)] - \right. \\
&\quad \left. - \frac{r\alpha\gamma_{2\kappa}}{\mu_{2\kappa}} [A_{5\kappa n} J'_n(\gamma_{2\kappa} r) + A_{6\kappa n} Y'_n(\gamma_{2\kappa} r)] \right\} \cos n\varphi e^{i(-\omega t + \gamma_p z)} \\
u_{q\kappa} &= - \sum_{n=0}^{\infty} \frac{n}{r} \left\{ \gamma_{\kappa} [A_{1\kappa n} J_n(\gamma_{1\kappa} r) + A_{2\kappa n} Y_n(\gamma_{1\kappa} r)] + \frac{r\gamma_{2\kappa}}{n} [A_{3\kappa n} J'_n(\gamma_{2\kappa} r) + A_{4\kappa n} Y'_n(\gamma_{2\kappa} r)] - \right. \\
&\quad \left. - \frac{r}{\mu_{2\kappa}} [A_{5\kappa n} J_n(\gamma_{2\kappa} r) + A_{6\kappa n} Y_k(\gamma_{2\kappa} r)] \right\} \sin n\varphi e^{i(-\omega t + \gamma_p z)}; \\
u_{z\kappa} &= \sum_{n=0}^{\infty} \left\{ \gamma_{\kappa} [A_{1\kappa n} J_n(\gamma_{1\kappa} r) + A_{2\kappa n} Y_n(\gamma_{1\kappa} r)] + \frac{\gamma_{2\kappa}^2}{n} [A_{5\kappa n} J_n(\gamma_{2\kappa} r) + A_{6\kappa n} Y_n(\gamma_{2\kappa} r)] \right\} \cos n\varphi e^{i(-\omega t + \gamma_p z)},
\end{aligned} \tag{9}$$

where

$$\begin{aligned}
\gamma_{1k}^2 &= \bar{\mu}_{1k}^2 - \gamma^2; \quad \bar{\mu}_{1k} = \frac{\omega}{a_{1k} \Gamma_k}; \quad \gamma_{2k}^2 = \bar{\mu}_{2k}^2 - \gamma^2; \\
\bar{\mu}_{2k} &= \omega / c_{sk} \Gamma_{2k}; \quad c_{sk}^2 = \frac{\bar{\mu}_k}{\rho_k}; \quad \gamma = m\pi / l, \quad (m=1,2,\dots) \quad (k=1,2,3)
\end{aligned}$$

To determine the arbitrary constants  $A_{1\kappa n}, A_{2\kappa n}, A_{3\kappa n}, A_{4\kappa n}, A_{5\kappa n}, A_{6\kappa n}$  used the boundary conditions (3). Then we obtain a system of homogeneous algebraic equations  $6\kappa + 3$  unknown and  $6\kappa + 3$  equations. A necessary and sufficient condition for the existence of solutions of this system, is the vanishing of its determinant. The order of the primary identifier  $(6\kappa + 3 \times 6\kappa + 3)$  and elements are expressed in terms of Bessel and Neumann functions of  $n$ -th order complex argument. This equation gives the dispersion equation for the dissipative systems.

Under the dispersion characteristics are understood according to the phase ( $C = C_R + iC_I$ ) and group velocity  $V = V_R + iV_I$  the wave number ( $\gamma_p$ ) while various parameters of the mechanical system. It is known that the magnitudes of  $C$  and  $V$  related to the value of the root of the dispersion equation

$$\Delta(C, \omega, \gamma_p, \lambda) = 0, \tag{10}$$

where  $\omega$ - complex frequency;  $\gamma_p$ -wave number,  $\lambda$  – wavelength. The phase and group velocity is related to the root meaning of the dispersion equation some complex dependencies. Thus, to be able to calculate the dispersion characteristics, it is necessary to make a study of the roots of the

equation (10) at the points of the complex plane, and also to develop a method of numerical determination.

The work for the solution of the transcendental equation (10) applies the method of Mueller, at each iteration of the method applied by Muller Gaussian with the release of the main element. Thus, the solution of equation (10) does not require disclosure of the determinant. As an initial approximation we choose the phase velocity of the waves corresponding elastic system. For complex roots Muller method simplifies the calculations and provides faster convergence [13,14].

### 3. THE PROPAGATION OF TRANSVERSE WAVES IN AN INFINITELY LONG CYLINDRICAL SHELL, LOCATED IN AN ELASTIC MEDIUM

The main objective of the research study of the existence of the phase velocity of propagation of the geometric and physical and mechanical parameters of the system. The basic equations of the theory of elasticity for such tasks are reduced to a plane problem ( $u_r = u_z = 0$ ). The components of the displacement vector in the cylinder and its environment are represented as:

$$u_{\theta 1} = -\frac{\partial \psi_1}{\partial r} = \left[ \left( A^I \frac{d}{dr} H_0^{(1)}(\bar{K}_1 r) + B^I \frac{d}{dr} H_0^{(2)}(\bar{K}_1 r) \right) \right] e^{-K_z z} ;$$

$$u_{\theta 2} = -C^I \frac{d}{dr} K_0(\bar{K}_2 r) e^{-iK_z z} ;$$

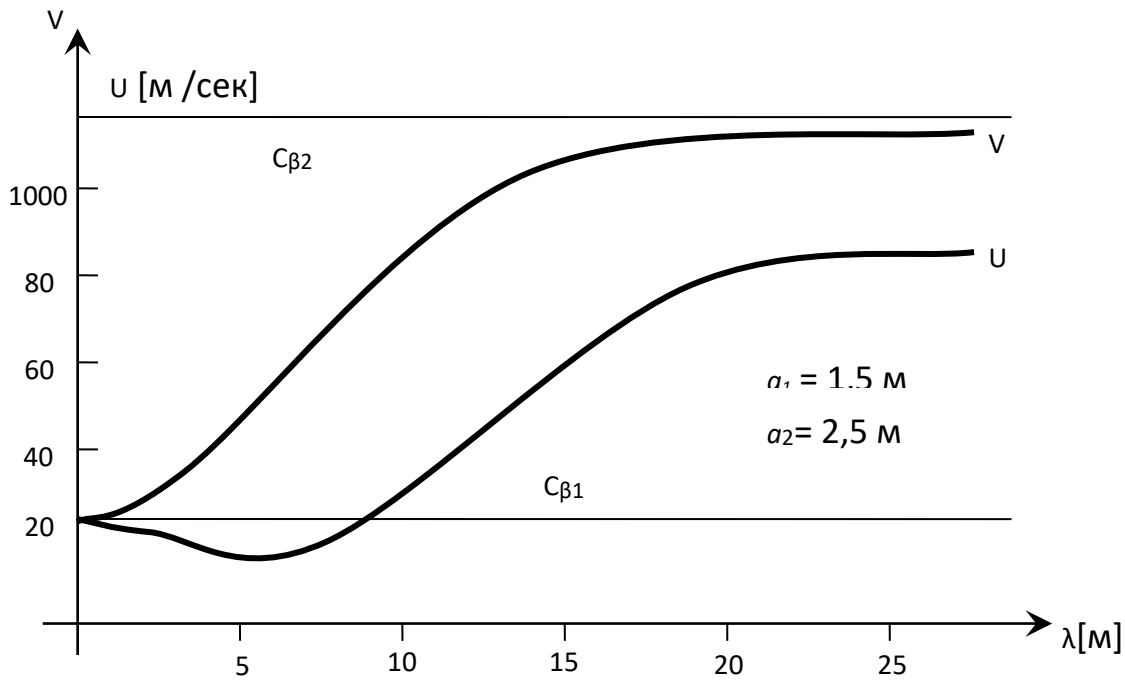
$$\gamma_{R\theta 1} = - \left\{ \begin{aligned} & \left[ A^I \frac{d^2}{dr^2} H_0^{(1)}(\bar{K}_1 r) + B^I \frac{d^2}{dr^2} H_0^{(2)}(\bar{K}_1 r) \right] e^{-K_z z} + \\ & + \frac{1}{r} \left[ A^I \frac{d}{dr} H_0^{(1)}(\bar{K}_1 r) + B^I \frac{d}{dr} H_0^{(2)}(\bar{K}_1 r) \right] \end{aligned} \right\} e^{-iK_z z}.$$

To determine the arbitrary constants  $A^1$ ,  $B^1$  and  $C^{11}$  It used the boundary conditions  $r = a_1$ :  $\sigma_{R\theta 1} = 0$  and  $r = a_2$   $u_{\theta 1} = u_{\theta 2}$ ,  $\sigma_{R\theta 1} = \sigma_{R\theta 2}$  we obtain a homogeneous system of algebraic equations of the third order. Their conditions of existence of nontrivial solutions we obtain the following dispersion equation:

$$\begin{vmatrix} -\bar{K}_1^2 H_0^{(1)}(\bar{K}_1 a_1) & -\bar{K}_1 H_0^{(2)}(\bar{K}_1 a_1) & 0 \\ -\bar{K}_1 H_1^{(1)}(\bar{K}_1 a_2) & -\bar{K}_1 H_0^{(2)}(\bar{K}_1 a_2) & -\bar{K}_{21} K_1(\bar{K}_{21} a_2) \\ -\bar{K}_1^2 \mu_1 H_0^{(1)}(\bar{K}_1 a_{21}) & -\bar{K}_1^2 \mu_1 H_0^{(2)}(\bar{K}_1 a_2) & -\bar{K}_2 \mu_2 K_0(\bar{K}_2 a_2) \end{vmatrix} = 0, \quad (11)$$

$$K_1 = \sqrt{K_1^2 - K_z^2}; \quad K_1^2 = \frac{\omega^2}{C_{s1}^2}; \quad \bar{K}_2^2 = \frac{\omega^2}{C_{s2}^2}; \quad C_{s1}^2 = \frac{\mu_1}{\rho_1}, \quad C_{s2}^2 = \frac{\mu_2}{\rho_2}$$

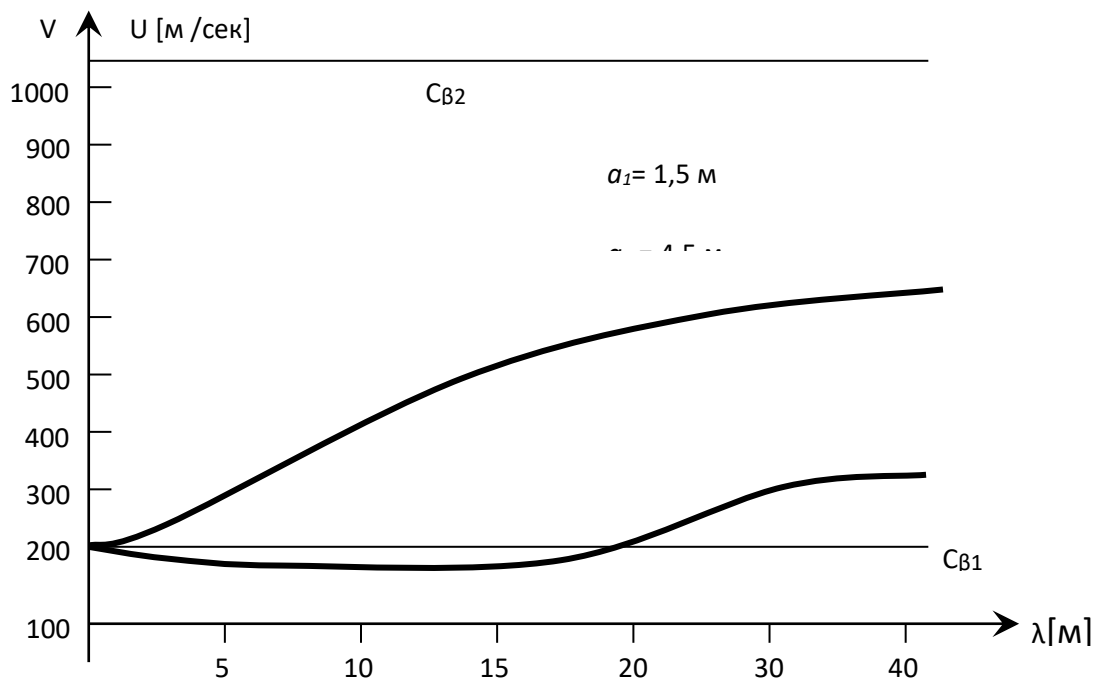
when  $K_2^2 > K_1^2$ .



**Figure 2** Changing the phase and group velocities, depending on the wavelength.

$$C_{\beta 1} = 200 \text{ m/s}; \quad C_{\beta 2} = 1100 \text{ m/s}$$

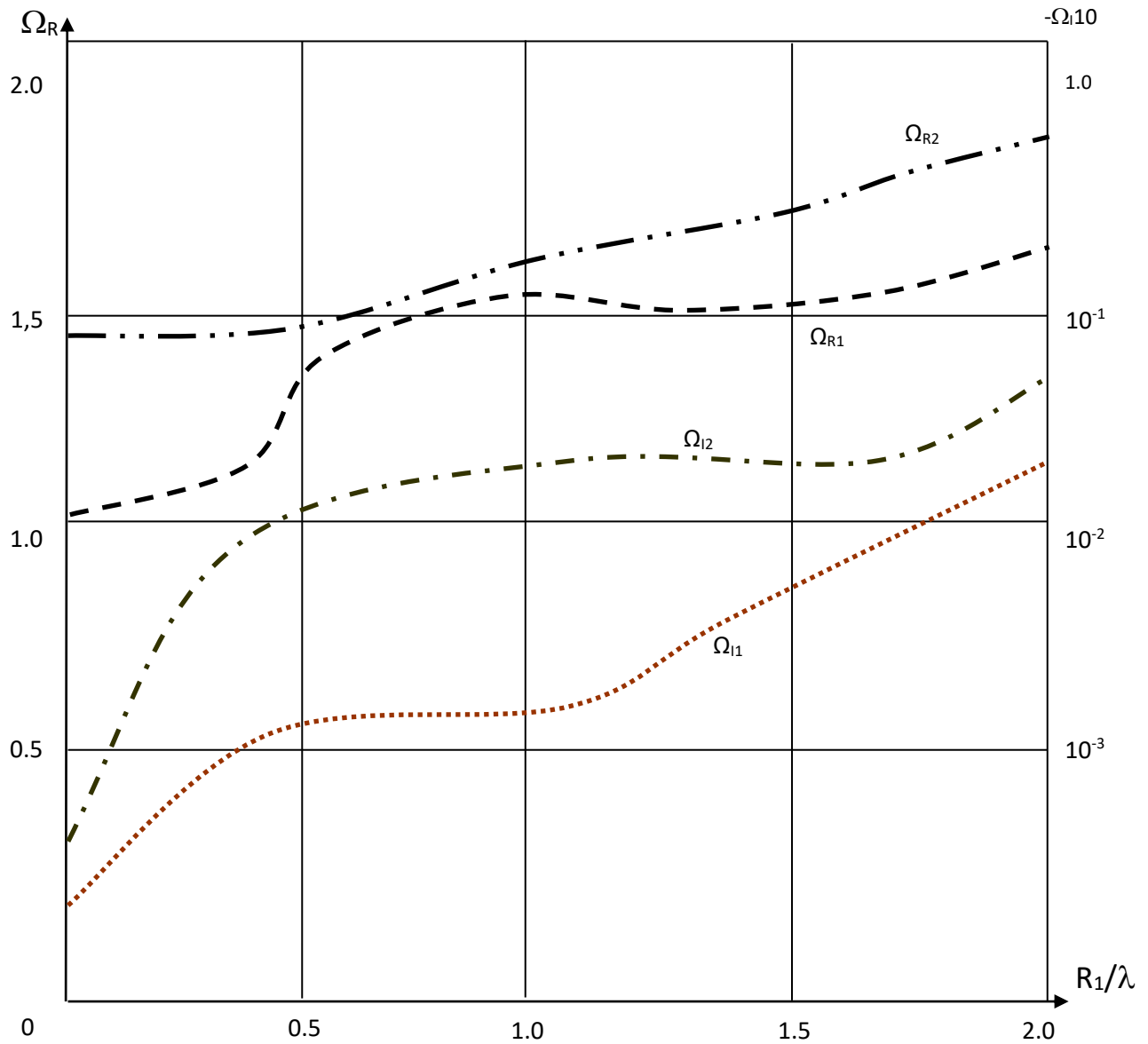




**Figure 3** Changing the phase and group velocity as a function wavelength from

The numerical results are shown in Figure 2. and Figure 3. Note that an increase in the thickness of the layer of the first and second mode phase velocity gradually decreases. From the figures it can be seen that the phase and group velocity in the area of wavelengths in the range of  $C_{\beta 1} < \lambda < C_{\beta 2}$ . In the of short waves phase velocity may be less than the velocity of shear waves.

**Distribution of natural waves in the three-layered cylindrical body** (Figure 1). Internal ( $r=a_1$ ) and external ( $r=a_4$ ) the surface of the three-layer cylinder is free from stress. Then the order of the determinant of the system of homogeneous algebraic equations (18x18), which is the dispersion equations ( $a_2 - a_1 = \Delta h_1$ ,  $a_3 - a_2 = \Delta h_2$ ,  $a_4 - a_3 = \Delta h_3$ ).

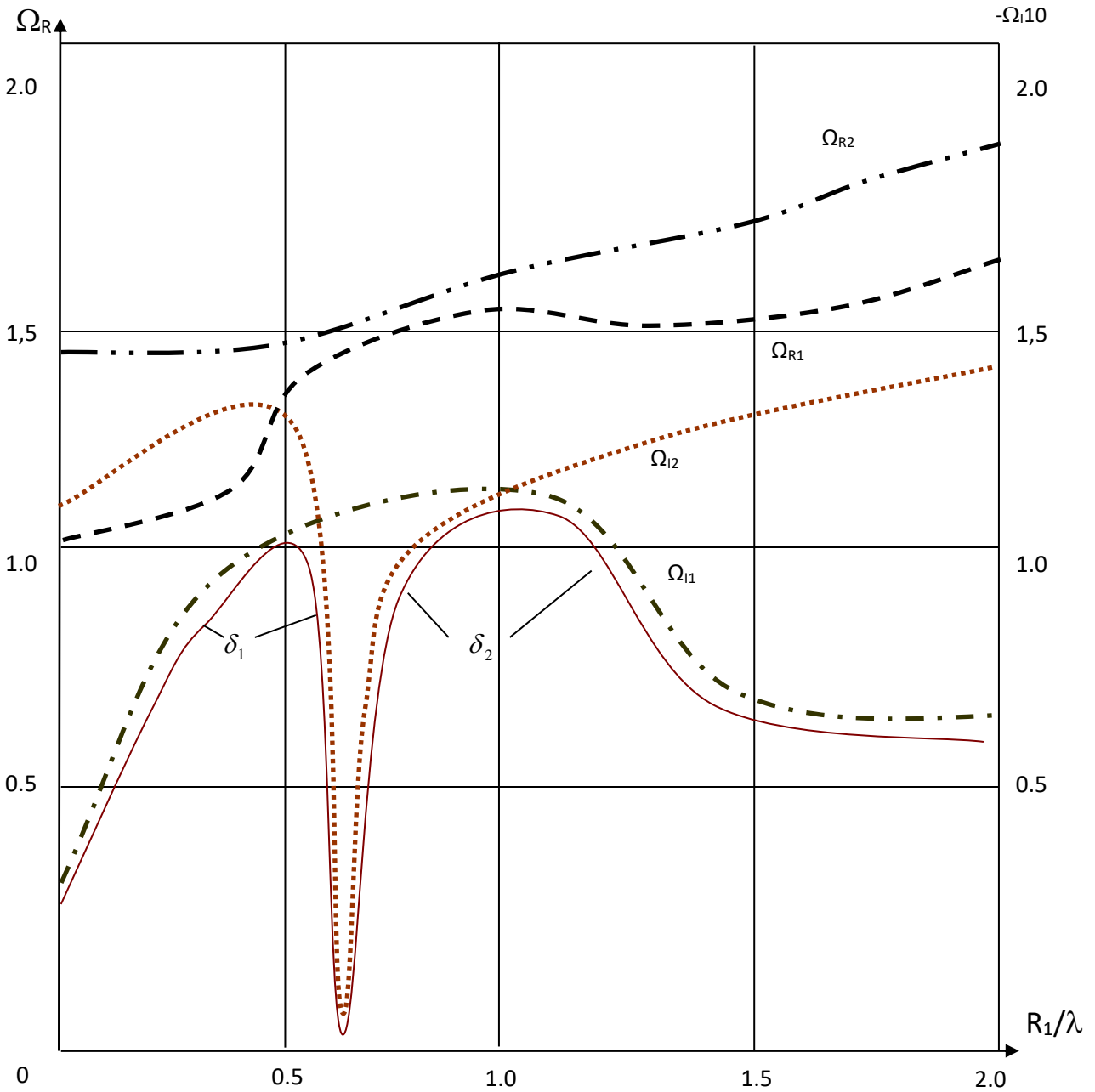


**Figure 4a** Changing the complex natural frequencies of the wave number (Dissipative homogeneous mechanical system).

In the calculations we take the following values:

$\Delta h_1 = 0,02 \text{ m}$ ,  $\Delta h_2 = 0,03 \text{ m}$ ,  $\Delta h_3 = 0,02 \text{ m}$ ,  $E_1 = 5,5 \cdot 10^8 \text{ Pa}$ ,  $E_2 = 3,5 \cdot 10^6 \text{ Pa}$ ,  $E_3 = 5,5 \cdot 10^8 \text{ Pa}$ ,  $\rho_1 = 27 \text{ kg/m}^3$ ,  $\rho_2 = 11 \text{ kg/m}^3$ ,  $\rho_3 = 27 \text{ kg/m}^3$ ,  $A = 0,048$ ;  $\beta = 0,05$ ;  $\alpha = 0,1$ .

Consider two options for a dissipative system. In a first embodiment, dissipative system is structurally homogeneous.



**Figure 4b** Changing the complex natural frequencies of the wave number (Dissipative heterogeneous mechanical system).

The dimensionless wave number  $\gamma_p^*$  it varies between 0 - 2. The calculation results are shown in Figure 4 as well. The dependence of the real and imaginary parts of the phase velocity of the dimensionless wave number  $\gamma_p^*$  It was monotonous, and dependence is the same for the real and imaginary part of the phase velocity. In the second embodiment, a dissipative system is structurally inhomogeneous, i.e. Rheological properties of the core layer ( $n = 2$ ) is equal to zero, the other parameters the same as adopted above. The calculation results are shown in Figure 4b. The dependence of the real part of the phase velocity of the dimensionless wave number of  $\gamma_p^*$  It is the same as for the homogeneous system: corresponding curves coincide up to 5%. Dependence of the imaginary part of the phase velocity of the  $\gamma_p^*$  nonmonotonic it turned out.

Of particular interest is the minimum value  $\xi$  a fixed damping coefficient:

$$\delta_{\omega} = \min(-\omega_{jk}), \delta_c = \min(-c_{jk}) \quad k = 1, 2, \dots, K, (10)$$

here  $\delta_c$  – coefficient determining the damping properties of the system.

For a homogeneous system coefficient  $\delta_c$  entirely determined by the imaginary part of the first modulo complex phase velocity. For inhomogeneous systems as a factor  $\delta_c$  can act as an imaginary part of the first and the second frequency depending on their values. "Turn the Tables" occurs when the characteristic value  $\gamma_p^*$ , while the value of the real parts of the first and second frequencies are closest.

Coefficient  $\delta_{\omega}$  and  $\delta_c$  at the specified characteristic value has a pronounced maximum.

#### 4. CONCLUSIONS

1. In the non-monotonic dependence of the imaginary part of the solution to the problem of wave propagation in inhomogeneous media dissipativno- discovered natural frequencies (or imaginary part of the phase velocity) of physical - mechanical and geometric parameters of the system.
2. It was found that the phase velocity of the highest forms of extension and torsion waves exceed the highest possible speed with the waves in an infinite medium, the group velocity never exceeds

S.Takzheutanavili that nondispersive medium grupovogo rate of 10-15% previschaet comparison dispersion medium. In other words, the shape of the pulses as they propagate not remain the same as in homogeneous elastic bodies.

3. Increase in viscosity reduces the real and imaginary parts of the phase velocity (or real and imaginary parts of the natural frequencies) of up to 20% for the homogeneous dissipative mechanical systems. For inhomogeneous dissipative mechanical systems increase in viscosity reduces the real part of the phase velocity (or the real part of the natural frequencies) to about 18%, and their imaginary parts, changed radically.

## REFERENCE

1. Vestyak A. Gorshkov A.G, Tarlakovsky D.V. Unsteady interaction of deformable bodies with the environment. - Results of science and technology. Fracture Mechanics. T. IIV, 1983. №4, p. 69-148.
2. Deyvis R.M.. Stress waves in solids. -Moscow. Foreign Literature Publishing House, 1961, -104 p.
3. Safarov I.I.. Oscillations and waves in dissipative nedorodnyh environments and structures. Tashkent. science, 1992-250 p.
4. Safarov I.I.. Dynamic viscoelastic vibration isolation systems with distributed parameters // seismodynamics buildings. Tashkent: Science, 1985, p. 134 - 149.
5. Safarov I.I., Ahmedov M.Sh., Boltaev Z.I.. Oscillations and diffraction of waves on the cylindrical body of the viscoelastic medium. Lambert Academic Publishing (Germany). 2016 262r. <http://dnb.d-nb.de>. ISBN: 978-3-659-67583-6 (monograph).
6. Koltunov M.A. Creep and relaksatsiya. M.: Higher School, 1976.- 277 p.
7. Koltunov M.A., Mayboroda V.P., Zubchaninov V.G.. Strength calculations of products from polymeric materialov.-M: Mechanical Engineering, 1983.-239 with
8. V.V. Bolotin, Yu. Novichkov.. The mechanics of multilayered structures. -M.: Engineering, 1980 - 375 p.
9. Bazarov M.B., Safarov I.I., Shokin Y.M.. Numerical simulation of vibrations dissipative heterogeneous and homogeneous mechanical systems. -Novosibirsk, Siberian Branch of the Russian Academy of Sciences, 1996. -189 p.
10. Kayumov S.S., Safarov I.I. Propagation and diffraction of waves in a dissipative-neodnorodnyh cylindrical deformable mechanical sistemah.-Tashkent; Fan 2004, -215 p.
11. Safarov I.I., Akhmedov M.Sh, Boltaev Z.I. Proper waves in layered media. Lambert Academic Publishing (Germany). 2016 192s. <http://dnb.d-nb.de>. ISBN: 978-3-659-87687-5 (monograph).
12. Grinchenko V.T., V.V. Maleshko. Harmonic Waves in elastic bodies.-K.: Naukova Dumka, 1981, -283p.
13. Safarov I.I, Boltaev Z. I., Akhmedov M. Sh. Properties of wave motion in a fluid-filled cylindrical shell / LAP, Lambert Academic Publishing. 2016 -105 p.
14. Safarov I.I, Akhmedov M. Sh., Boltaev Z.I. Natural oscillations and diffraction of waves on the cylindrical body. Lambert Academic Publishing (Germany). 2016. 245r. <http://dnb.d-nb.de>. ISBN: 978-3-659-93556-5 (monograph).